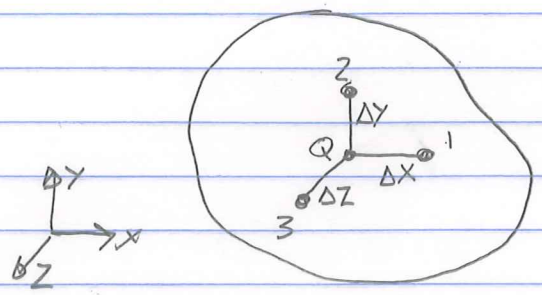
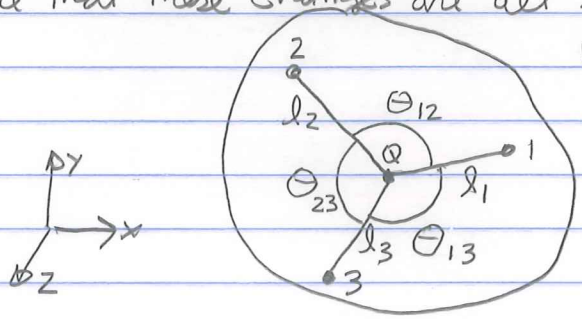
4. Strain definition

Strain is a measure of deformation. Consider an undeformed (unloaded) body:



where lines Q-1, Q-2 and Q-3 are of lengths ΔX, ΔY and ΔZ and are parallel to x, y and z, respectively.

As a result of the deformation, the lengths of the 3 lines and the angles between them change. We will assume here that these changes are all small and that the overall rotation is also small.



Line lengths are ℓ_1, ℓ_2 and ℓ_3 and angles are θ_{12}, θ_{23} and θ_{13}.

Six strains are defined by a limiting process as follows,

$\lim_{\Delta X \to 0} \frac{\ell_1 - \Delta X}{\Delta X} = \varepsilon_x$, the normal (axial) strain in the x direction at Q

$\lim_{\Delta Y \to 0} \frac{\ell_2 - \Delta Y}{\Delta Y} = \varepsilon_y$, the normal (axial) strain in the y direction at Q

$\lim_{\Delta Z \to 0} \frac{\ell_3 - \Delta Z}{\Delta Z} = \varepsilon_z$, the normal (axial) strain in the z direction at Q

$\lim_{\Delta X, \Delta Y \to 0} \left(\frac{\pi}{2} - \theta_{12}\right) = \gamma_{xy}$, the shear strain in the x-y plane at Q

$\lim_{\Delta Y, \Delta Z \to 0} \left(\frac{\pi}{2} - \theta_{23}\right) = \gamma_{yz}$, the shear strain in the y-z plane at Q

$\lim_{\Delta X, \Delta Z \to 0} \left(\frac{\pi}{2} - \theta_{13}\right) = \gamma_{xz}$, the shear strain in the x-z plane at Q

These quantities are sufficient to determine the strains with respect to any oriented set of axes.

Also, given the strains as functions of x, y and z for a body, the displacements as functions of x, y and z can be determined. (Also need other information to specify the overall location and orientation of the body.)

Finally, the strains, when defined as above, can be related to the stresses (which actually cause the strains) through equations that involve material properties. That is, given the material properties, the stress state can be determined from the strain state, and vice versa. This is the subject of the next section.